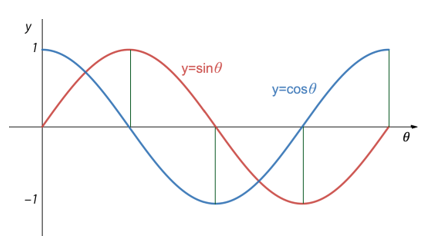
# 5.5 Amplitude and Period of the Sine and Cosine Functions

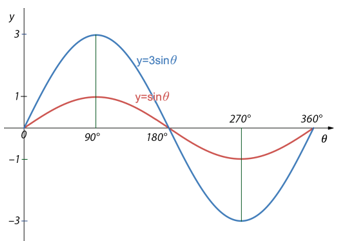
## Amplitude

We have seen how the graphs of both the sine function, and the cosine function , oscillate between and . That is, the heights oscillate between –1 and 1.

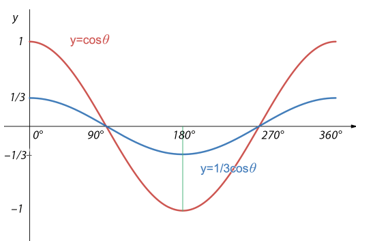


The height from the horizontal axis to the peak (or through) of a sine or cosine function is called

the **amplitude** of the function. Each of the curves and has amplitude 1.



If we were to multiply the sine function  
by , getting ,  
each of the sine values would be multiplied by 3 making each value 3 times what it was. Each height would be tripled.  
The amplitude of is 3.



If we were to multiply the cosine function by , getting ,  
each of the cosine values would be multiplied by 1/3 making each value 1/3 of what it was. Each height of would be 1/3 of what it was.  
The amplitude of is 1/3.

## THE AMPLITUDE OF AND

Suppose represents a positive number. Then the **amplitude** of both and is and it represents height from the horizontal axis to the peak of the curve.

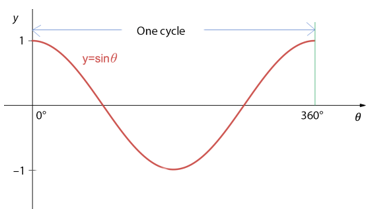
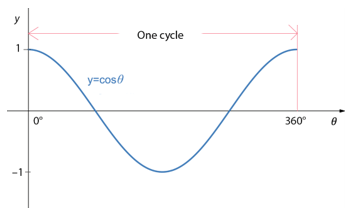
Examples

The amplitude of is 5/8. This means that the peak of the curve is 5/8 of a unit above the horizontal axis.

The amplitude of is 3. This means that the peak of the curve is 3 units above the horizontal axis.

## Period

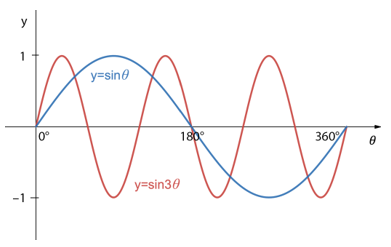
Both the sine function and cosine function, and go through exactly one cycle from 0° to 360°. The **period** of the sine function and cosine functions, and is the “time” required for one complete cycle.

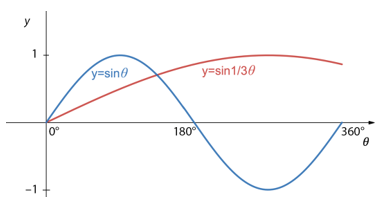
An interesting thing happens to the curves and when the angle is multiplied by some positive number, If the number is greater that 1, the number of cycles on 0° to 360° increases for both and . That is, the peaks of the curve are closer together, meaning their periods decrease. If the number is strictly between 0 and 1, the peaks of the curve are farther apart, meaning their periods increase.

## THE PERIOD OF AND

Suppose represents a positive number. Then the **period** of both and is As B gets bigger, gets smaller and the period increases.



If we were to multiply the angle in the sine function by , getting , each of the angle’s values would be multiplied by 3 making each value 3 times what it was. Each angle would be tripled and there would be 3 cycles in the interval 0° to 360°.  
The period of is .  
The period of is smaller than that of .



If we were to multiply the angle in the sine function by , getting Each of the angle’s values would be multiplied by 1/3 making each value 1/3 what it was and there would be only 1/3 of a cycle in the interval 0° to 360°.  
The period of  
 is = .  
The period of  
 is greater than that of

## Using Technology

We can use technology to help us construct the graph of a sine or cosine function.

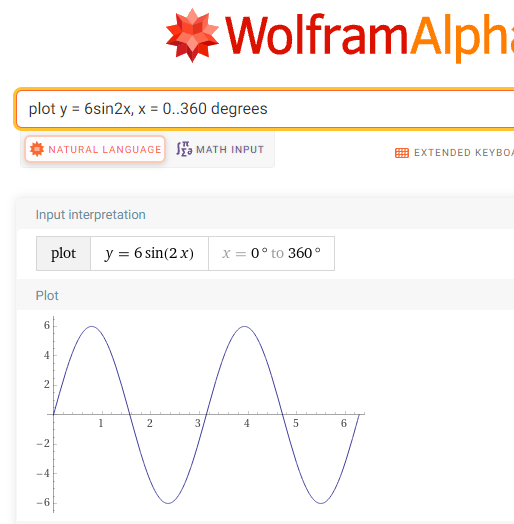
Go to www.wolframalpha.com.

Plot two complete cycles of from 0° to 360°.

Example (1)

Type plot y = 6sin2x, x = 0..360 degrees in the entry field.

WolframAlpha tells you what it thinks you entered, then produces the graph.



You can see that WolframAlpha has plotted two complete cycles from 0° to 360° with amplitude 6.

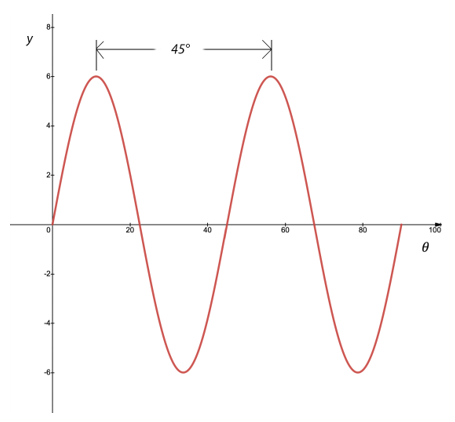
Find the period of .

Example (2)

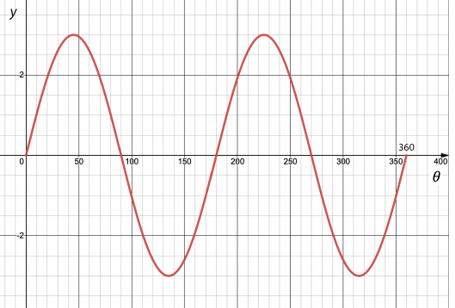
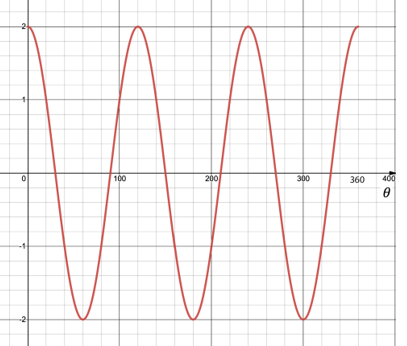
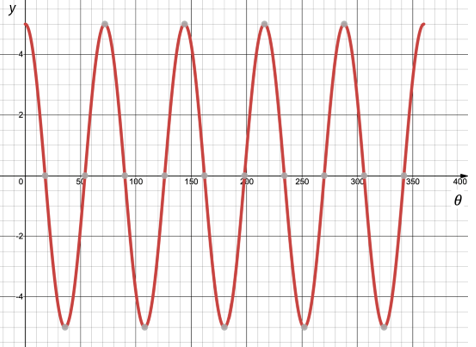
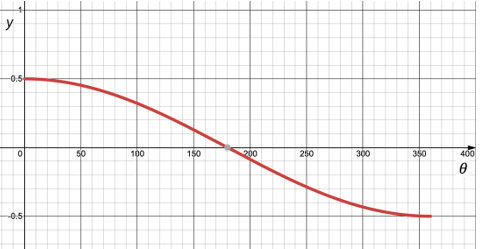
We just need to evaluate with .

The period of is

The graph of helps us visualize this 45° period. You can see that the peaks differ by 45°.



## EXAMPLES

1. Write the equation of each graph.
2.  ANS:
3.  ANS:
4.  ANS:
5.  ANS:
6. How many complete cycles are there in the graph of from 0° to 360°? What is the period and amplitude of this function?

ANS: 3 complete cycles. Period is Amplitude is 4.

1. How many complete cycles are there in the graph of from 0° to 360°? What is the period and amplitude of this function of

ANS: of a complete cycle. Period is . Amplitude is 5.

1. Write the equation of a sine curve that has amplitude 15 and period 50°. You need to specify both in . Keep in mind that the period of this function is .

ANS: , where

1. Write the equation of a cosine curve that has amplitude 100 and period 12°. You need to specify both in . Keep in mind that the period of this function is .

ANS: , where

1. Write the equation of a cosine function that has amplitude 3 and makes two complete cycles from 0° to 180°.

ANS: We need to specify both in . Since the amplitude is 3 Since the curve makes two complete cycles from 0° to 180°, it must make 4 complete cycles from 0° to 360°. So,

1. Write the equation of a sine function that has amplitude 4 and makes three complete cycles from 0° to 90°.

ANS: We need to specify both in . Since the amplitude is 4 Since the curve makes three complete cycles from 0° to 90°, it must make 12 complete cycles from 0° to 360°. So, .

## NOTE TO THE INSTRUCTOR

First, consider presenting the **amplitude** of the sine and cosine function.

Ask what would happen if we multiplied by 4.

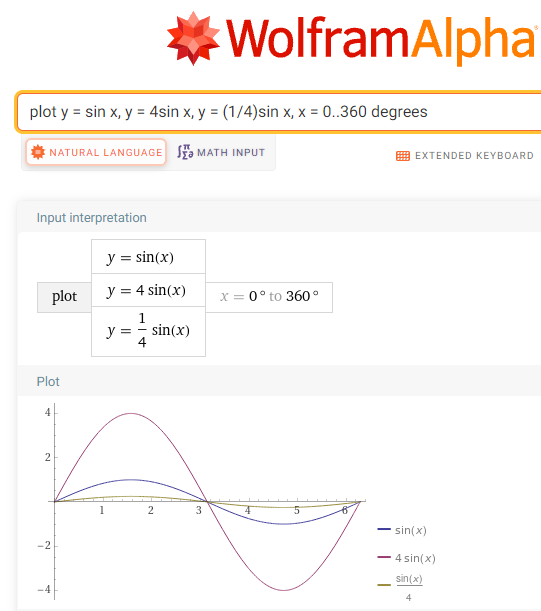
If we were to multiply the sine function by , getting , each of the sine values would be multiplied by 4 making each value 4 times what it was. Each height would be quadrupled. The amplitude of is 4.

Now discuss what would happen if we multiplied by 1/4.

If we were to multiply the sine function by , getting , each of the sine values would be multiplied by , making each value of what it was. Each height of would be of what it was in . The amplitude of is .

To compare the graphs, use [WolframAlpha](http://www.wolframalpha.com/) or [Desmos](https://www.desmos.com/calculator) to construct the graphs of

, , and all on the same coordinate system.



Now present the **period** of the sine and cosine function.

Suppose represents a positive number. Then the period of both and is As B gets bigger, gets smaller and the period increases.

Ask what would happen if we were to multiply the angle by

If we were to multiply the angle in the sine function by , getting , each of the angle’s values would be multiplied by 4 making each value 4 times what it was. Each angle would be quadrupled and there would be 4 cycles in the interval 0° to 360°. The period of is . The period of is smaller than that of .

Ask what would happen if we were to multiply the angle is multiplied by

If we were to multiply the angle in the sine function by , getting Each of the angle’s values would be multiplied by 1/4 making each value 1/4 what it was and there would be only 1/4 of a cycle in the interval 0° to 360°. The period of is = . The period of is greater than that of

For comparison, you could use WolframAlpha or Desmos to construct the graph of each of these functions on the same coordinate system.

[5-5\_amplitude\_and\_period\_of\_sine\_and\_cosine\_functions.docx](https://oer4cte.org/math4gamedevelopers/teacher/5-5_amplitude_and_period_of_sine_and_cosine_functions.docx), attributed to Denny Burzynski (author) and Downey Unified School District (publisher) is licensed under CC BY-NC 4.0. To view a copy of this license, visit https://creativecommons.org/licenses/by-nc/4.0